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SUMMARY

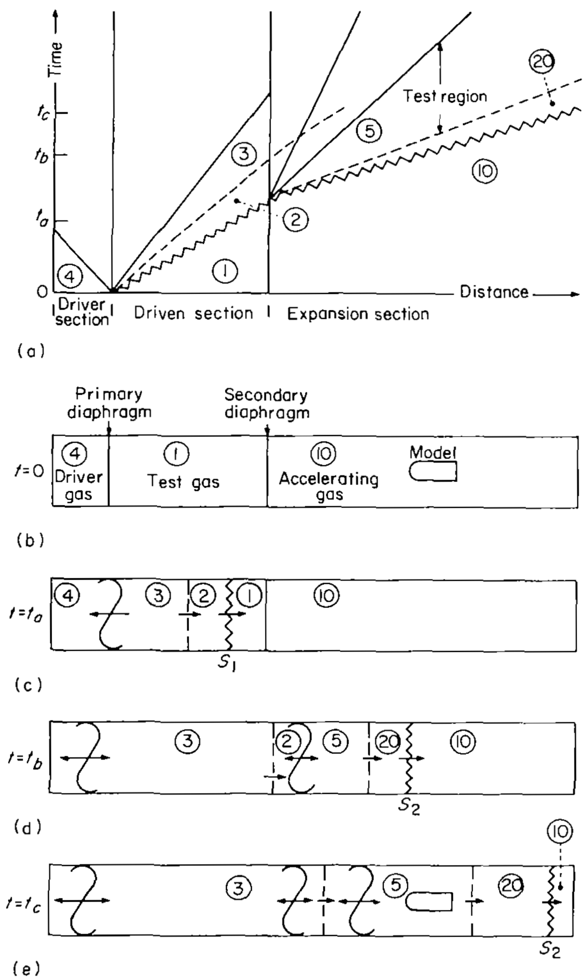
Charts are presented which facilitate the theoretical analysis of certain flow processes involving an unsteady expansion wave in equilibrium real air. The parameters evaluated are related to (a) the velocity change in an unsteady expansion wave, (b) the maximum velocity attainable in a shock-wave—expansion-wave cycle, (c) the time for characteristic traversal through an expansion wave, and (d) the entropy in shock-initiated flows. Such parameters are of particular interest in analysis of the shock-initiated expansion-tube cycle.

INTRODUCTION

Evaluation of the steady isentropic expansion of equilibrium real air is relatively simple because of the invariance of entropy and total enthalpy in this process. It is necessary only to follow a constant-entropy line on a Mollier diagram to determine the fluid properties, and the fluid velocity is subsequently determined by equating the kinetic energy with the difference between the total and local enthalpies. However, in an isentropic unsteady expansion, the evaluation of an integral is necessary because the total enthalpy is not constant. This paper presents the results of such integrations in the form of working charts suitable for facile interpolation.

The work reported herein resulted from the requirement for rapid analysis of expansion-tube design and performance. (See ref. 1.) The expansion-tube cycle is illustrated in figure 1 by a distance-time diagram. Note that the flow expands unsteadily between region 2 and region 5, the test region. Calculation of the flow properties in the test region necessitates an intermediate integration across the unsteady expansion fan.

Reference 2 tabulates some functions for an unsteady expansion, but the tables are based on a constant value of the specific heat ratio γ. An approximation for



(a) Distance-time plot.

(b) Prior to run, t = 0.

(c) After primary diaphragm burst, $t = t_a$.

(d) After secondary diaphragm burst, $t = t_b$.

(e) During testing, $t = t_c$.

Figure 1.- Sketches illustrating expansion-tube cycle.

real air in equilibrium may be found in reference 3 for some of the functions required, but the entropy intervals are too large for satisfactory interpolation and the enthalpy range is somewhat limited, especially at higher entropy levels.

Although originally constructed for application to the expansion-tube cycle, the utility of the charts is by no means restricted solely to this purpose.

SYMBOLS

a speed of sound

a_o speed of sound in argon-free air at reference conditions, 1,089 ft/sec = 0.3320 mm/μsec

h static enthalpy

$l = \int_0^p \frac{dp}{\rho a}$

M_S shock Mach number, U_S/a_1

p static pressure

R gas constant for argon-free air, 1.724×10^3 ft^2/(sec^2)($^\circ$R) = 6.886×10^{-2} Btu/(lb)($^\circ$R)

s entropy per unit mass

T absolute temperature

T_o temperature at reference conditions, 491.69° R = 273.16° K

t time

U_S shock velocity

u flow velocity

u_l limiting velocity

x distance

γ ratio of specific heats

γ_o ratio of specific heats at reference conditions, 1.4

$$\eta = \log_e \frac{(at^2)_*}{(at^2)_{h/RT_o}}$$

ρ density

Subscripts:

1,2,5 flow regimes in expansion-tube cycle (fig. 1)

$*$ conditions at $h/RT_o = 1$

ANALYSIS

Velocity Change in One-Dimensional Isentropic Unsteady Expansion

The differential equation for an isentropic unsteady expansion is

$$du = \mp \left(\frac{dh}{a}\right)_{s/R} \tag{1}$$

with the negative and positive signs referring to upstream and downstream waves, respectively. The equation for the speed of sound at reference conditions

$$a_o^2 = \gamma_o RT_o \tag{2}$$

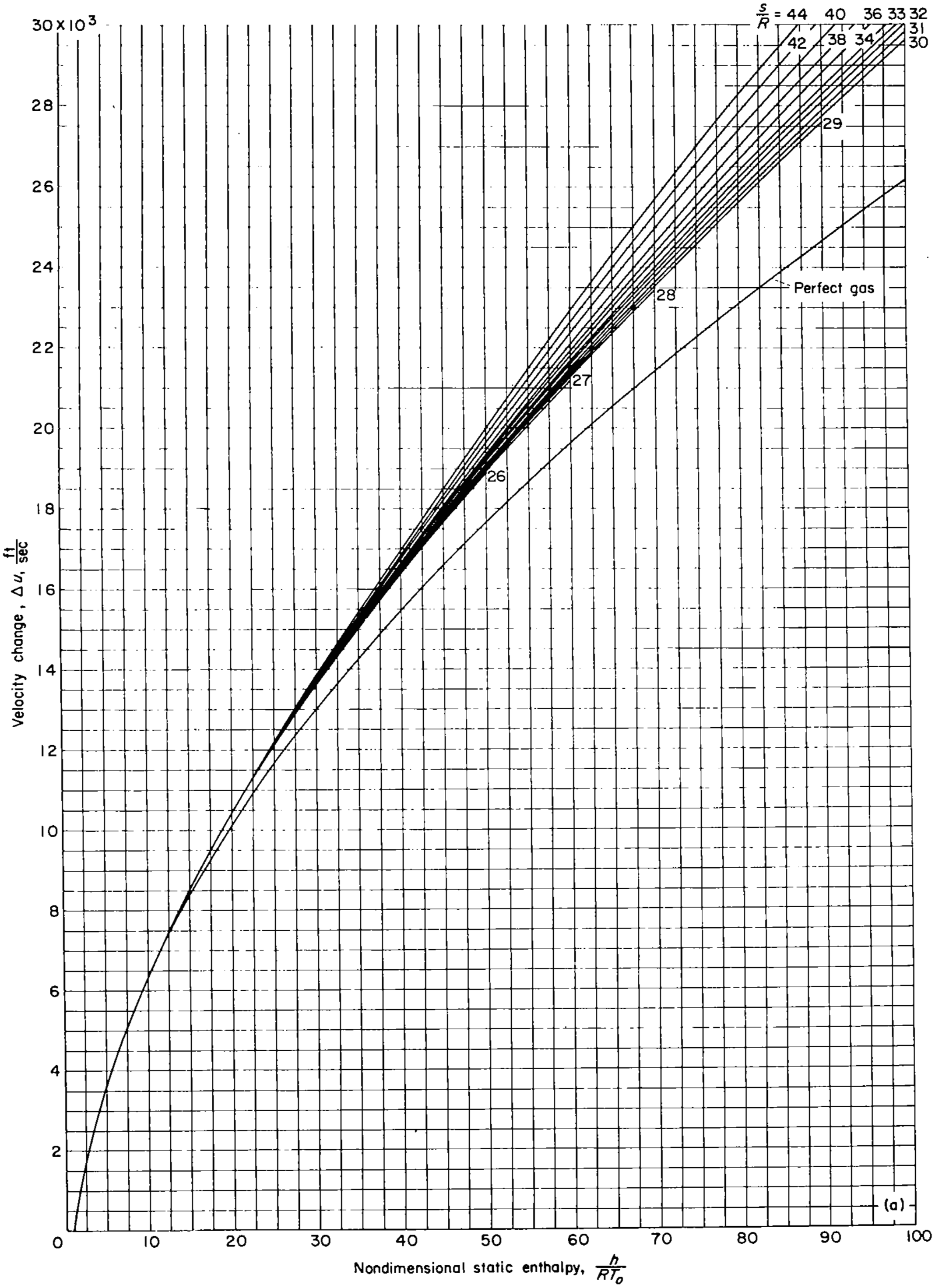
is used to put equation (1) in the following dimensionless form:

$$d\,\frac{u}{a_o} = \mp \frac{1}{\gamma_o}\left(\frac{d\,\dfrac{h}{RT_o}}{a/a_o}\right)_{s/R} \tag{3}$$

The equilibrium properties of real air from references 4, 5, and 6 were used to plot the variation of $(a/a_o)^{-1}$ with h/RT_o for various unit values of s/R. Equation (3) was then integrated numerically from the limit $h/RT_o = 1$ to an arbitrary value of h/RT_o. The function $\Delta u = f(h/RT_o)$ is defined as

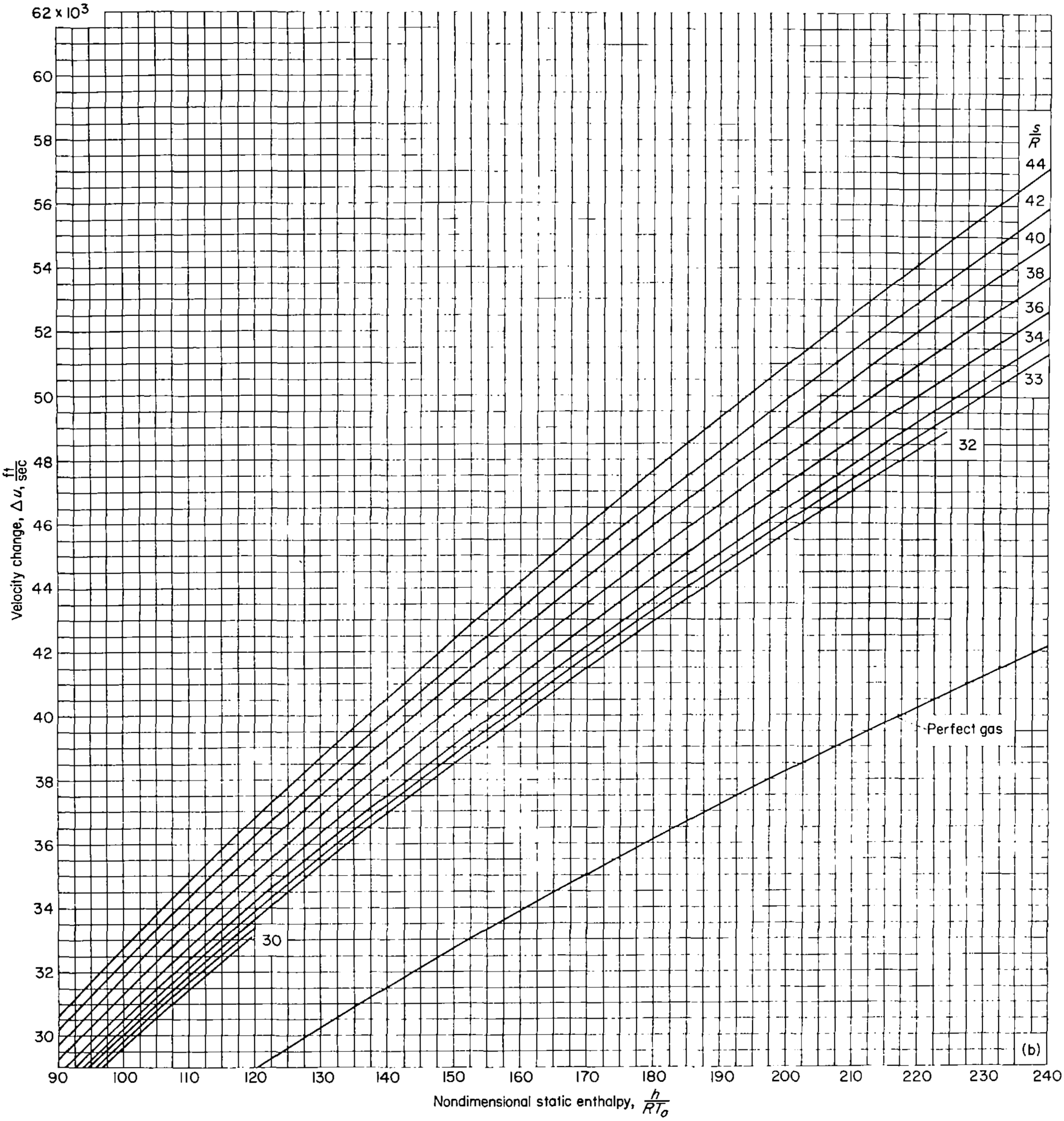
$$\Delta u\left(\frac{h}{RT_o}\right) = u_* - u\left(\frac{h}{RT_o}\right) = \mp \frac{a_o}{\gamma_o} \int_{h/RT_o=1}^{h/RT_o} \frac{d\,\dfrac{h}{RT_o}}{a/a_o} \tag{4}$$

Values of Δu as a function of h/RT_o are plotted in figure 2. The parameter Δu is analogous to the Prandtl-Meyer angle ν of steady flow (ref. 7).



(a) $0 \leq \frac{h}{RT_o} \leq 100$.

Figure 2.- Velocity change in a one-dimensional isentropic unsteady expansion as a function of static enthalpy.



(b) $90 \leqq \frac{h}{RT_o} \leqq 240$.

Figure 2.- Concluded.

The quantity $\nu(M)$ is the angle through which the flow is turned in an isentropic expansion from $M = 1$ by waves of a single family to reach the Mach number M; and $\Delta u(h/RT_o)$ is the velocity change obtained by single-family isentropic expansion from arbitrary h/RT_o down to $h/RT_o = 1$. The value of Δu is positive for upstream (Q) expansion waves and negative for downstream (P) expansion waves.

The value of Δu for a perfect gas with constant specific heat may be found in closed form by integrating equation (4). The result is

$$\Delta u = 2 \frac{a_o}{\gamma_o}\sqrt{\frac{\gamma_o}{\gamma_o - 1}}\left(\sqrt{\frac{h}{RT_o}} - 1\right) \tag{5}$$

This perfect-gas value is also plotted in figure 2 and falls below the real-air values.

To illustrate the application of figure 2, consider an expansion (Q type) between static-enthalpy levels h_2 and h_5 (where $h_2 > h_5$). Entering figure 2 at these values of enthalpy at the appropriate value of s/R yields

$$\left.\begin{aligned} \Delta u_2 &= u_* - u_2\left(h_2/RT_o\right) \\ \Delta u_5 &= u_* - u_5\left(h_5/RT_o\right) \end{aligned}\right\} \tag{6}$$

The difference between the two expressions is the velocity increment imparted to the flow by an isentropic unsteady expansion between enthalpies h_2 and h_5, namely

$$u_5 - u_2 = \left(u_5 - u_*\right) - \left(u_2 - u_*\right) = \Delta u_2 - \Delta u_5 \tag{7}$$

A function corresponding to Δu is the l of reference 3, which is defined as the integral at constant entropy:

$$l = \int_0^p \frac{dp}{\rho a} \tag{8}$$

Thus

$$\frac{l}{a_o} = \frac{\Delta u\left(\frac{h}{RT_o}\right)}{a_o} - \frac{\Delta u(0)}{a_o} \tag{9}$$

Although reference 3 is adequate for many purposes, the use of an approximate model for equilibrium air, along with the fact that results are tabulated only at intervals of 5 in s/R and that the range of enthalpies is somewhat restricted, limits its suitability for the rapid analysis of the expansion-tube cycles. In ranges where comparison of the results of reference 3 with those of the present paper was possible, agreement within 2 percent was found for the corresponding velocity functions.

Transit Time of Downstream Characteristic Through a Centered Upstream Expansion Wave

Figure 13 of reference 1 indicates that one of the factors limiting the maximum available testing time in region ⑤ is the arrival of a P-characteristic wave. This P-wave is the first of a family generated either by reflection of the expansion wave (④ → ③) in the driver or by the interaction of the first contact surface and the expansion fan (② → ⑤). (See ref. 8.)

The speed at which the P-characteristic propagates is

$$\frac{dx}{dt} = u + a \tag{10}$$

Now, for a centered upstream (Q) wave originating at $x = 0$ and $t = 0$,

$$x = (u - a)t \tag{11}$$

Differentiating equation (11) and substituting the result into equation (10) produces the relation

$$\frac{du}{a} = 2\frac{dt}{t} + \frac{da}{a} \tag{12}$$

Combining equations (12), (1), and (2) yields

$$2\frac{dt}{t} + \frac{da}{a} = -\frac{1}{\gamma_o}\left[\frac{d\,\frac{h}{RT_o}}{(a/a_o)^2}\right]_{s/R} \tag{13}$$

which may then be integrated as

$$\eta = \frac{1}{\gamma_o}\int_{h/RT_o=1}^{h/RT_o}\frac{d\,\frac{h}{RT_o}}{(a/a_o)^2} \tag{14}$$

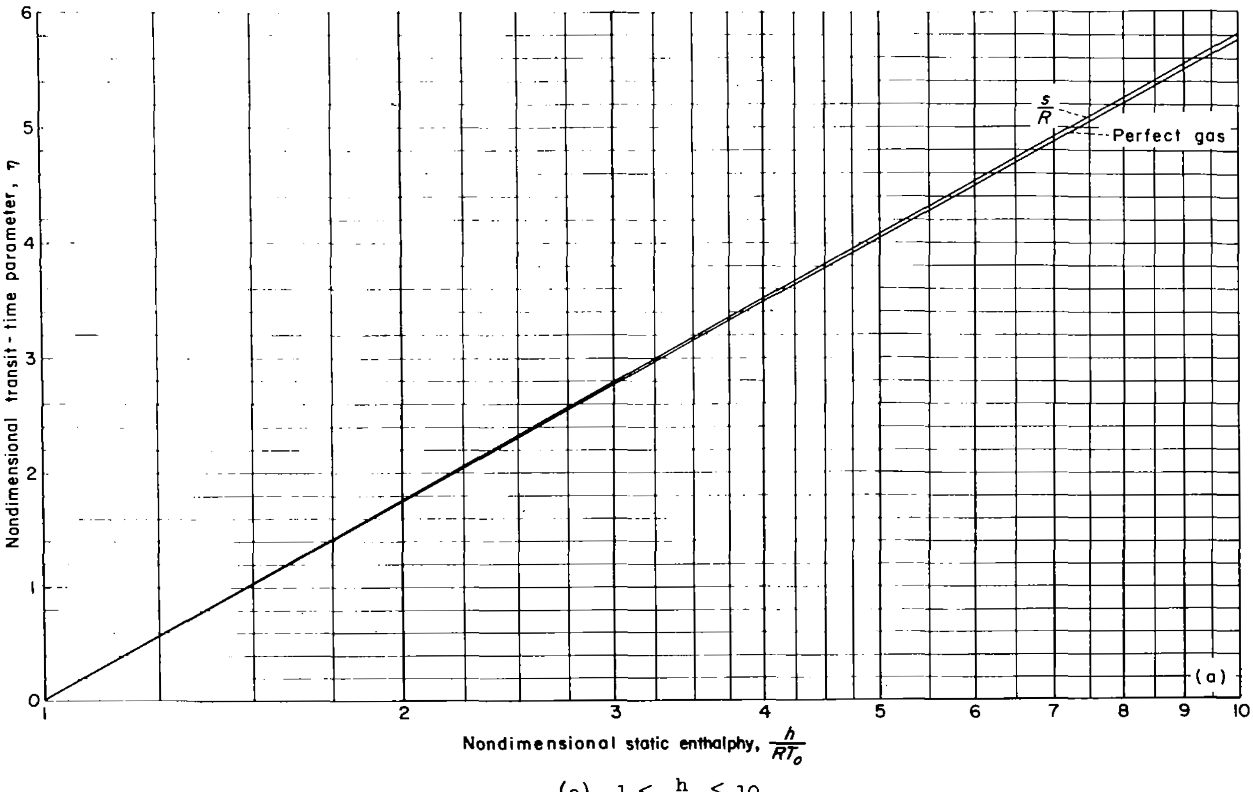
where η, the nondimensional transit-time parameter, is defined as

$$\eta = \log_e \frac{(at^2)_*}{(at^2)_{h/RT_o}} \tag{15}$$

Plots of $(a/a_o)^{-2}$ as a function of h/RT_o were numerically integrated between limits of arbitrary h/RT_o and $h/RT_o = 1$. In addition, integration of equation (14) for a perfect gas with constant specific heat yields

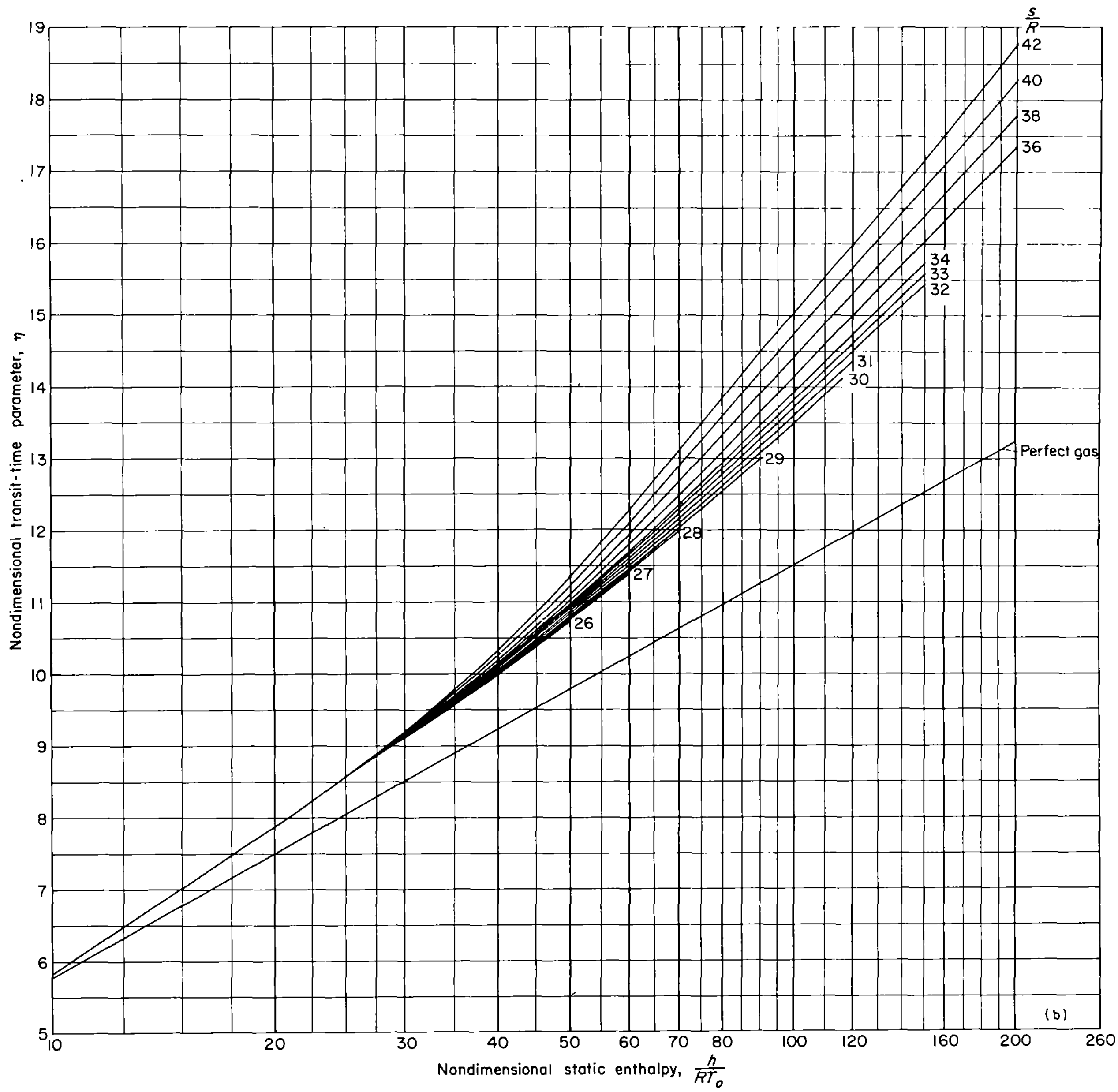
$$\eta = \frac{1}{\gamma_o - 1} \log_e \frac{h}{RT_o} \tag{16}$$

The results of these integrations are presented in figure 3. This chart is used in the manner prescribed for figure 2, but in addition it requires that the



(a) $1 \leq \frac{h}{RT_o} \leq 10$.

Figure 3.- Transit-time parameter of downstream characteristics through a centered upstream expansion wave as a function of static enthalpy. RT_o = 33.89 Btu/lb.



(b) $10 \leqq \frac{h}{RT_o} \leqq 200.$

Figure 3.- Concluded.

sound speed at the two limits of integration be known in order to determine the transit time. Also, it must be remembered that zero time refers to the time at the origin of the expansion fan.

Entropy of Shock-Initiated Flows

In applying the results of figures 2 and 3 to shock-initiated flows, it is necessary first to determine the entropy of the flow. Figure 4 is a plot of shock velocity against flow entropy for various values of pressure p_1 ahead of the shock when $T_1 = 300^\circ$ K. The shock-speed scale is expressed in mm/μsec to permit easy use of the charts of reference 4 for determining the pressure and velocity in state ② behind the shock wave. For values of p_1 between 0.001 and

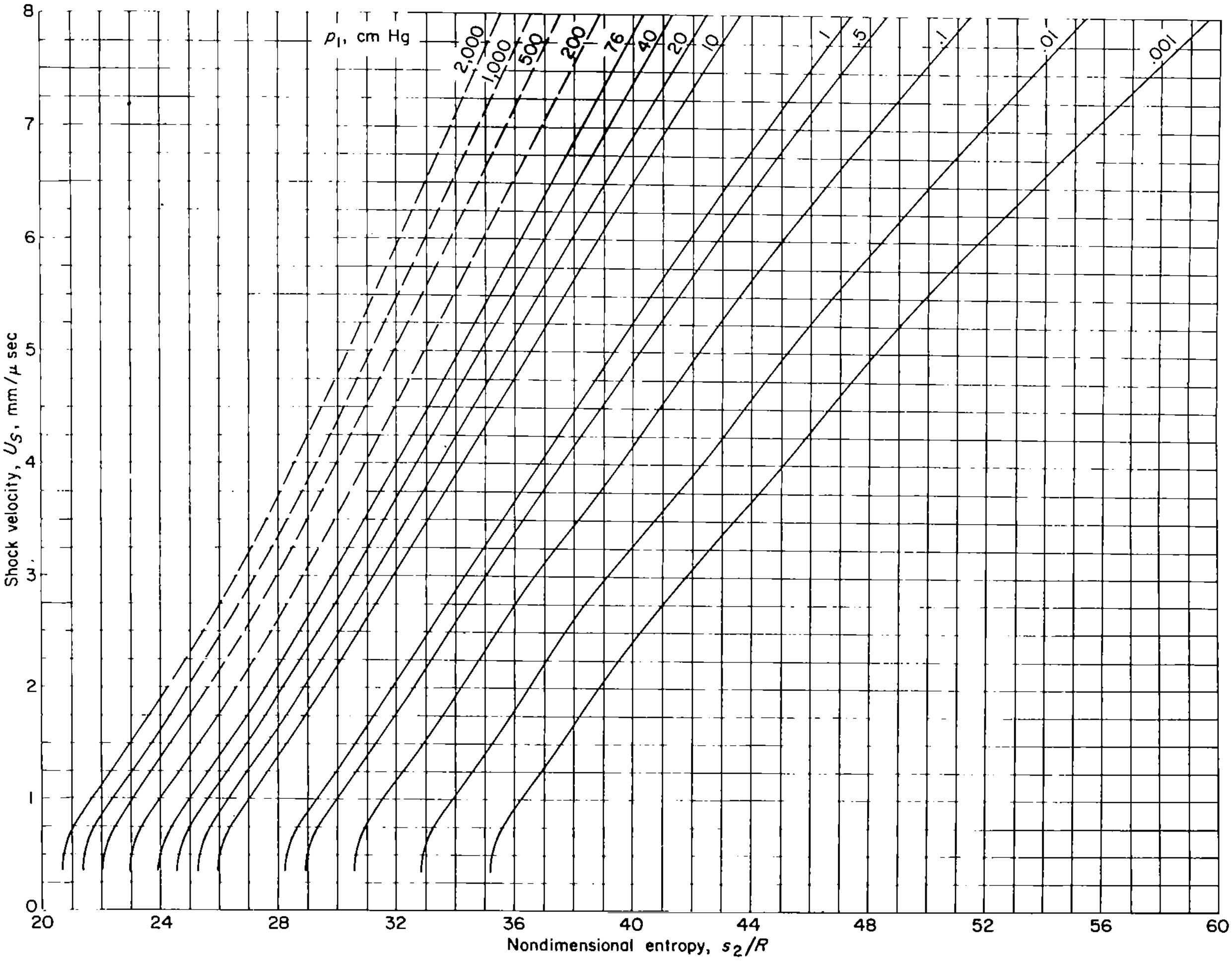


Figure 4.- Entropy of shock-initiated air flows as a function of shock velocity. $T_1 = 300^\circ$ K.

76 cm Hg and $U_S > 2.1$ mm/μsec the value of s_2/R was read from Mollier charts by using the data of reference 4, which encompass only this range of p_1 and U_S. For values of $U_S < 2.1$ mm/μsec $\left(M_S < 6.05\right)$ the equilibrium results of reference 9 were employed to plot the variation of $\dfrac{s_2 - s_1}{R}$ with U_S, and these plots were then interpolated at the desired pressure level $\left(p_1\right)$. The dashed curves of figure 4 for $p_1 \geqq 200$ cm Hg are extrapolations obtained from a cross plot of $\dfrac{s_2 - s_1}{R}$ against $\log p_1$ at constant values of U_S. The largest error $\left(\text{at } U_S = 8 \text{ mm}/\mu\text{sec}, \; p_1 = 2{,}000 \text{ cm Hg}\right)$ in this extrapolation is believed to be less than 0.2 in s_2/R.

Limiting Velocity of Shock-Initiated Flows

In the analysis of the expansion-tube cycle or other shock-initiated flows it is often advantageous to know the limiting velocity of the flow. This limiting velocity is defined for the purposes of this paper as the velocity that a flow might attain if expanded isentropically by an unsteady expansion wave to $h/RT_o = 1$; that is, $u_l \equiv u + \Delta u(h/RT_o)$. Of course, this limiting velocity is not the maximum velocity obtainable, since an additional increment would result from further expansion from $h/RT_o = 1$ to $h/RT_o = 0$. For the particular case of a shock-initiated flow the limiting velocity becomes

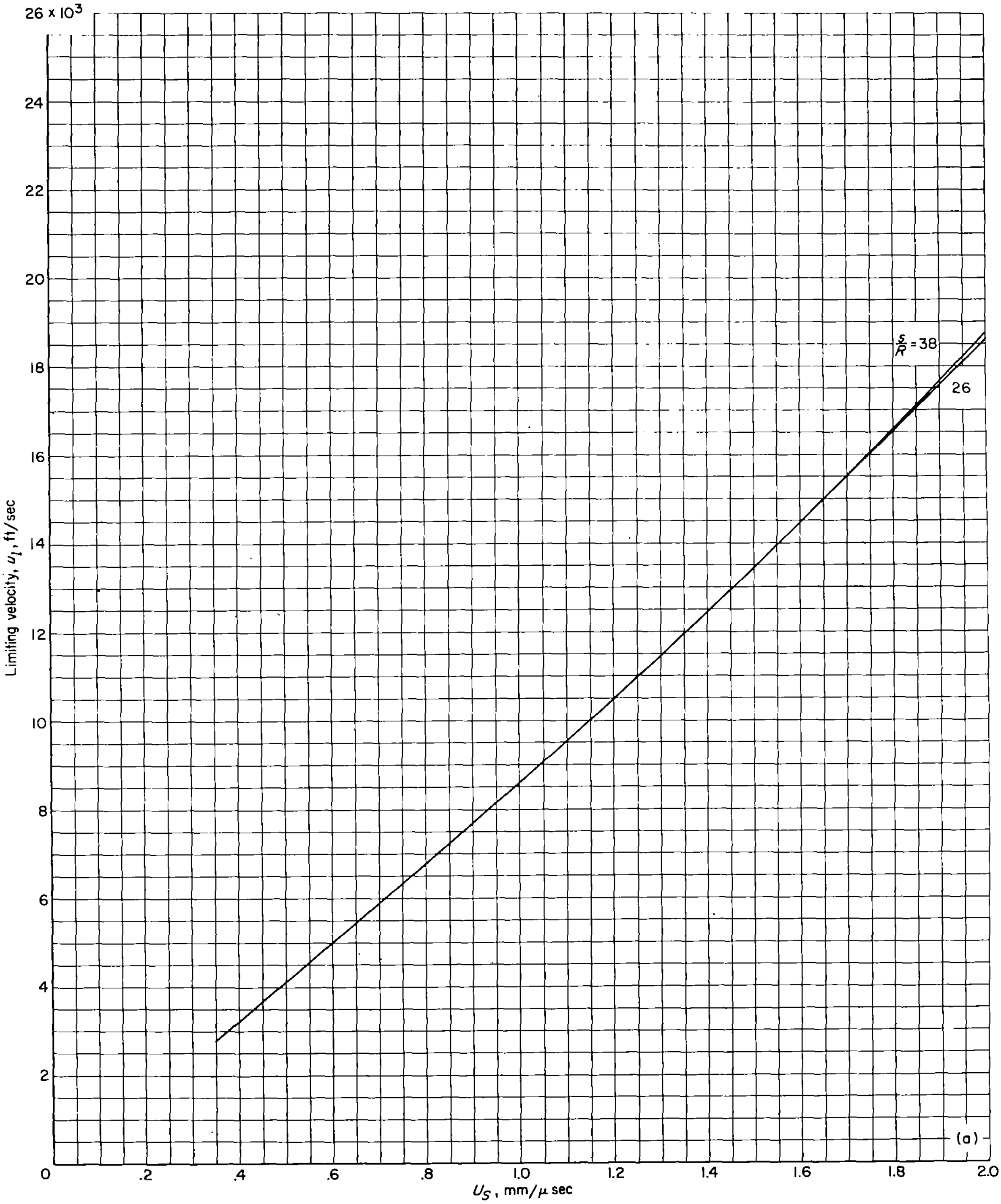
$$u_l = u_2 + \Delta u(h_2/RT_o) \tag{17}$$

Since both u_2 and Δu are functions of the shock speed and flow entropy, u_l can be plotted against U_S with s_2/R as a parameter; figure 5 is such a plot. It is of interest to note the very high values of u_l attainable.

DISCUSSION

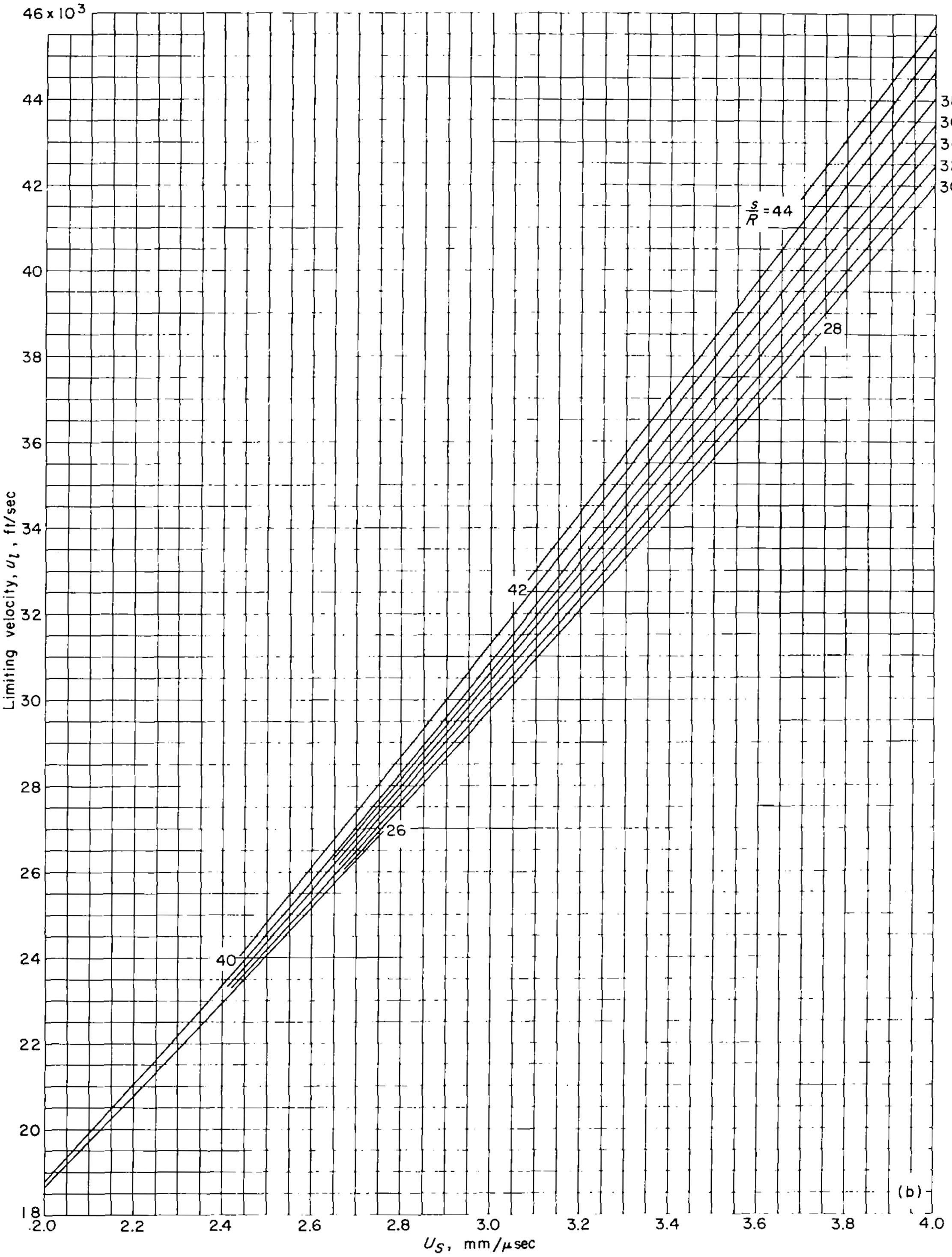
The utilization of the curves of this report is illustrated by the following example. The problem is the determination of the conditions required in an expansion-tube cycle to produce a test velocity of $u_5 = 40{,}000$ ft/sec at an altitude of 250,000 feet. The free-stream nondimensional entropy s/R at this altitude is 33.0 and the enthalpy $h/RT_o = 2.3$. Since state 5 is produced by an unsteady expansion from state 2, the entropy and limiting velocity are equal in both states. First $u_{l,5}$ is computed by reading $\Delta u\left(h_5/RT_o\right)$ from figure 2:

$$u_{l,5} = u_5 + \Delta u\left(h_5/RT_o\right) = 40{,}000 + 1{,}500 = 41{,}500 \text{ ft/sec}$$



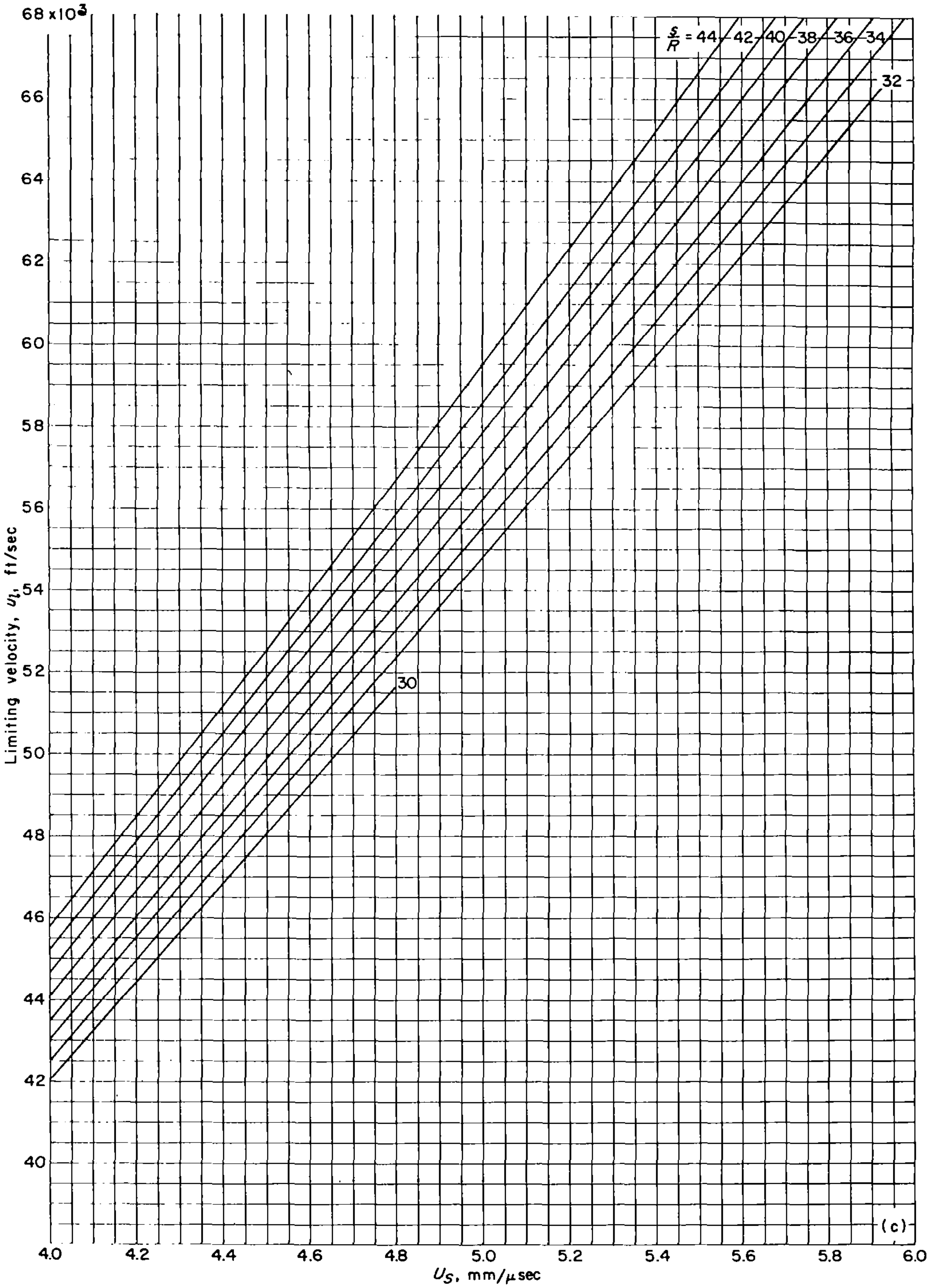
(a) $0 \leqq U_S \leqq 2.0$.

Figure 5.- Limiting velocity in shock-initiated air flows as a function of shock velocity. $T_1 = 300^\circ$ K.



(b) $2.0 \leqq U_S \leqq 4.0$.

Figure 5.- Continued.



(c) $4.0 \leqq U_S \leqq 6.0$.

Figure 5.- Concluded.

Then the shock speed for $u_7 = 41{,}500$ ft/sec at $s_2/R = 33.0$ is found from figure 5 to be $U_S = 3.90$ mm/μsec. This value of U_S is then used to enter figure 4 to determine $p_1 = 26$ cm Hg.

Once the shock speed U_S and initial pressure p_1 are known, the other properties in region 2 can be found from the charts of reference 4. Pertinent values for this example are $h_2/RT_o = 99.4$, $p_2/p_o = 54.7$, and $u_2 = 11{,}400$ ft/sec. The simplicity of this procedure is a contrast to the iteration method of reference 1.

The application of figure 3 in determining the ratio of transit times $\left(t_{II}/t_{III}\right.$ of fig. 13 in ref. 1$\left.\right)$ may be similarly found.

CONCLUDING REMARKS

Charts are presented which facilitate the theoretical analysis of certain flow processes involving an unsteady expansion wave in equilibrium real air. The parameters evaluated are related to (a) the velocity change in an unsteady expansion wave, (b) the maximum velocity attainable in a shock-wave—expansion-wave cycle, (c) the time for characteristic traversal through an expansion wave, and (d) the entropy in shock-initiated flows. Such parameters are of particular interest in analysis of the shock-initiated expansion-tube cycle, as illustrated by the sample computation.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., February 18, 1963.

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